# SOLVENCY II SYNDICATE SCR FOR 2015 YEAR OF ACCOUNT

## NOTES TO THE SUPPLEMENTARY QUESTIONNAIRE

JUNE 2014

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# PURPOSE

This document provides additional information on certain calculations and tests used in the Supplementary Questionnaire. It is a supporting document only and contains no additional requirements or guidance.

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# STANDARDISED CALCULATION OF THE POST-DIVERSIFICATION AMOUNTS ON FORM 309

**(Supplementary Questionnaire:** PartII_PostDiv_Inputs**)**

### Purpose

Beginning with the 2015 YOA LCR submission, agents will be required to use a standardised methodology for calculating the post-diversification values shown on form 309. This note provides a step-by-step illustration of how to apply the method. It does not contain any guidance or requirements that are not specified in the SCR Guidance or the Supplementary Questionnaire. It is for informational purposes only.

### Executive Summary

Form 309 provides columns for the post-diversified amounts of each SCR risk category. These post-diversified amounts are intended to represent the contribution of each risk category to the SCR.

In summary, the new methodology requires agents to do the following on each LCR submission on both a one-year and ultimate basis:

- Rank the simulated balance sheet positions (the SCR is the $VaR_{99.5}$ of the balance sheet position)
- Calculate a proxy “Confidence Interval SCR” (CI SCR) by averaging over a range of simulations specified by Lloyd’s
- Calculate the average amounts for each risk category over the same range of simulations used to determine the CI SCR
- Calculate the post-diversified amounts for each risk category by scaling the averages by the ratio of the syndicate’s selected post-diversified SCR to the proxy CI SCR
- Report the post-diversified amounts on form 309 columns C and G

The above methodology applies only to the calculation of the post-diversified SCR risk categories in rows 1-8 of columns C and G of form 309 **excluding** the SCR (row 9). The diversified SCR is to be calculated according to a method judged appropriate by the agent, consistent with the SCR Guidance 4.11 - 4.12. The “proxy CI SCR” is an intermediate value to be discarded after determining the post-diversified amounts for the SCR risk categories.

The range of simulations are defined to ensure that the “true” internal model $99.5^{th}$ percentile SCR (i.e. the value that the SCR converges to as simulation error approaches nil) lies within the range at a 95% confidence level. Details are in the Appendix.

The approach is based on outputs currently produced for the LCR. There should be no need for agents to revise their model structure or calculate new metrics.

Please contact Eric Pizarro (eric.pizarro@lloyds.com) or Adhiraj Maitra (adhiraj.maitra@lloyds.com) if you have further questions on the methodology or this memo.

## What agents must do: Simple numerical example

The purpose of following example is to illustrate the methodology; it is not intended to provide a realistic example of syndicate model outputs or the number of simulations required. The example is shown for the ultimate case (column G on form 309); the same methodology applies to the one-year case (column C on form 309).

Suppose the model was run for 10,000 simulations and the 1:200 pre-diversification amounts by risk category are as shown below. The ultimate SCR is 122.4m.

| | | | | Ultimate basis (Note 309.2) | |
|---|---|---|---|---|---|
| | | | | Pre diversification | Post diversification |
| | | | | GBP (m) | GBP (m) |
| | | | | E | G |
| **Insurance Risk** | total: | After diversification between Premium and Reserve Risk | 1 | 112.5 | |
| | split: | Premium Risk (see note above) | 2 | 80.9 | |
| | split: | Reserve Risk (Note 309.4) | 3 | 48.8 | |
| **Credit Risk** | total: | After diversification between Reinsurance and Other Credit Risk | 4 | 18.1 | |
| | split: | Reinsurance Credit Risk | 5 | 17.2 | |
| | split: | Other Credit Risk | 6 | 2.6 | |
| **Market Risk** (see note above) | | | 7 | 22.9 | |
| **Operational Risk** | | | 8 | 15.0 | |
| **TOTAL** (Note 309.3) | | | 9 | 168.5 | |
| **Diversification Credit - between risk categories** | | | 10 | - 46.1 | |
| **DIVERSIFIED TOTAL** (Note 309.3) | | | 11 | **122.4** | |

The steps to calculate the post-diversification amounts to be shown in column G are as follows. (The corresponding section in the Supplementary Questionnaire is referred to in brackets.)

1. **Rank the simulated balance sheet positions from smallest to largest.** (SuppQ PartII_PostDiv_Inputs Q1)

| Simulation No. | Premium | Reserve | Insurance | RI credit | Other credit | Credit | Market | Operational | Balance sheet position | Balance sheet position rank |
|---|---|---|---|---|---|---|---|---|---|---|
| | Risk Type | | | | | | | | | |
| 3526 | -61.4 | -93.5 | -154.8 | 0.1 | 0.0 | 0.1 | -22.1 | 2.0 | -174.9 | 1 |
| 239 | -63.9 | -80.5 | -144.4 | 0.0 | 0.1 | 0.1 | -16.8 | 0.0 | -161.2 | 2 |
| 7637 | -56.4 | -94.3 | -150.7 | 10.0 | 0.1 | 10.1 | -18.1 | 2.0 | -156.8 | 3 |
| 6284 | 87.3 | 109.8 | 197.2 | 10.0 | 0.0 | 10.0 | 1.3 | 0.0 | 208.4 | 9998 |
| 5101 | 169.5 | 26.2 | 195.7 | 10.0 | 0.0 | 10.0 | 12.4 | 0.0 | 218.1 | 9999 |
| 6584 | 166.3 | 63.7 | 230.0 | 0.0 | 0.1 | 0.1 | 14.0 | 0.0 | 244.0 | 10000 |

Simulations sorted based on size of simulated balance sheet position

2. **Determine the appropriate range of simulations for the post-diversification calculations from ranges provided by Lloyd's.** (SuppQ PartII_PostDiv_Inputs Q1) The simulation ranges have been selected to provide a 95% confidence interval for the "true" internal model SCR. See the Appendix for details on the methodology.

   Since 10,000 simulations have been run, the range for the post-diversification calculations would be from 9,937 to 9,964 *after* sorting by ascending size of the balance sheet position.

| Post-diversification calculations: specification of ranges | | | | | | | |
|---|---|---|---|---|---|---|---|
| No. simulations | **10,000** | 25,000 | 50,000 | 75,000 | 150,000 | 200,000 | 250,000 |
| SCR percentile | **99.5** | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 |
| Confidence level that SCR percentile lies in range | **95.00%** | 95.00% | 95.00% | 95.00% | 95.00% | 95.00% | 95.00% |
| Range definition in terms of rank of SCR simulations | | | | | | | |
| upper bound | **9,964** | 24,897 | 49,781 | 74,663 | 149,304 | 199,062 | 248,819 |
| lower bound | **9,937** | 24,854 | 49,720 | 74,588 | 149,197 | 198,939 | 248,682 |
| range width | **28** | 44 | 62 | 76 | 107 | 124 | 137 |

3. **Determine the proxy CI SCR and average values for each SCR risk type over the specified range of simulations.** (SuppQ PartII_PostDiv_Inputs Q2 & Q3 and PartII_PostDiv_Outputs M1)

   The specified range from simulation 9,937 to 9,964 is shown below.

| Simulation No. | Premium | Reserve | Insurance | RI credit | Other credit | Credit | Market | Operational | Balance sheet position | Balance sheet position rank | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Risk Type | | | | | | | | | | |
| 3526 | -61.4 | -93.5 | -154.8 | 0.1 | 0.0 | 0.1 | -22.1 | 2.0 | -174.9 | 1 | |
| 239 | -63.9 | -80.5 | -144.4 | 0.0 | 0.1 | 0.1 | -16.8 | 0.0 | -161.2 | 2 | |
| ..... | | | | | | | | | | | |
| 5242 | 81.2 | 29.9 | 111.1 | 0.0 | 0.5 | 0.5 | -9.2 | 0.0 | 114.7 | 9937 | Balance sheet position and risk types averaged over these simulations |
| 8306 | 69.9 | 44.4 | 114.3 | 10.0 | 0.0 | 10.0 | -9.3 | 0.0 | 115.0 | 9938 | |
| ..... | | | | | | | | | | | |
| 1413 | 43.6 | 71.6 | 115.2 | 0.0 | 0.0 | 0.0 | -2.8 | 15.0 | 127.4 | 9963 | |
| 736 | 17.4 | 90.7 | 108.1 | 0.1 | 0.1 | 0.1 | 12.5 | 7.5 | 128.2 | 9964 | |
| 5211 | 1.7 | 121.9 | 123.6 | 0.1 | 0.5 | 0.6 | -3.3 | 7.5 | 128.4 | 9965 | |
| ..... | | | | | | | | | | | |
| 5101 | 169.5 | 26.2 | 195.7 | 10.0 | 0.0 | 10.0 | 12.4 | 0.0 | 218.1 | 9999 | |
| 6584 | 166.3 | 63.7 | 230.0 | 0.0 | 0.1 | 0.1 | 14.0 | 0.0 | 244.0 | 10000 | |

For example, insurance risk would be averaged over the values 111.1, 114.3,…, 115.2, 108.1 = 116.0. This is just insurance risk averaged over the 28 simulations for which the rank of the balance sheet position falls within the defined range.

The proxy CI SCR would be the average of 114.7, 115.0, …,127.4, 128.2 = 122.8.

The post-diversified insurance risk would be 116.0 * (selected SCR)/(proxy CI SCR) = 116.0 * (122.4/122.8) = 115.6. This is the amount that would be shown as post-diversified insurance risk on form 309. The results for the other risks are shown below. The scaling factor ensures that their sum is equal to the selected diversified SCR of 122.4m shown in row 11 of column E of form 309.

| Form 309 reported SCR (m): | 122.4 | | |
|---|---|---|---|
| **SCR Risk Type** | **CI Value (m)** | **Scaling Factor** | **Post Diversified (m) (for Form 309 col G)** |
| Insurance | 116.0 | 99.6% | 115.6 |
| Premium | 74.0 | | 73.7 |
| Reserve | 42.0 | | 41.9 |
| Credit | 2.5 | | 2.5 |
| RI credit | 1.9 | | 1.9 |
| Other credit | 0.7 | | 0.7 |
| Market | 0.9 | | 0.9 |
| Operational | 3.4 | | 3.3 |
| CI SCR | 122.8 | | 122.4 |

**4. Populate rows 1-8 of col G of from 309 with the post-diversified amounts from step 3. Discard the CI SCR.** (PartII_PostDiv_Outputs M1)

| | | | | Ultimate basis (Note 309.2) | |
|---|---|---|---|---|---|
| | | | | Pre diversification | Post diversification |
| | | | | GBP (m) | GBP (m) |
| | | | | E | G |
| **Insurance Risk** | total: | After diversification between Premium and Reserve Risk | 1 | 112.5 | 115.6 |
| | split: | Premium Risk (see note above) | 2 | 80.9 | |
| | split: | Reserve Risk (Note 309.4) | 3 | 48.8 | |
| **Credit Risk** | total: | After diversification between Reinsurance and Other Credit Risk | 4 | 18.1 | 2.5 |
| | split: | Reinsurance Credit Risk | 5 | 17.2 | |
| | split: | Other Credit Risk | 6 | 2.6 | |
| **Market Risk** (see note above) | | | 7 | 22.9 | 0.9 |
| **Operational Risk** | | | 8 | 15.0 | 3.3 |
| **TOTAL** (Note 309.3) | | | 9 | 168.5 | |
| **Diversification Credit - between risk categories** | | | 10 | - 46.1 | |
| **DIVERSIFIED TOTAL** (Note 309.3) | | | 11 | **122.4** | |

The post-diversified values for reserve risk, premium risk, RI credit risk and other credit risk are reported on the Supplementary Questionnaire.

## Q&A

- **Q**. Why average over the range instead of taking individual values corresponding to the simulated 99.5th percentile balance sheet position?
    - **A.** Model outputs for the SCR risk types can vary considerably by simulation around a given percentile. The average will be more stable.
- **Q**. Why bother with the proxy CI SCR? Why not just use the selected diversified SCR?
    - **A.** The post-diversified SCR risk categories should sum to the selected diversified SCR. This will not normally be the case, unless the selected SCR is exactly equal to the proxy CI SCR. The CI SCR is therefore needed to scale the averaged SCR risk category amounts. In most cases the scaling factor should be small.
- **Q**. Is this approach required for other 1:200 values on the LCR, e.g. the pre-diversified amounts on form 309 or the claim amounts on Form 313?
    - **A.** No, there are currently no plans to require the approach to be applied to other parts of the LCR.
- **Q.** The LCR does not show the post-diversified amounts for premium risk and reserve risk, only for insurance risk in total. Are agents required to calculate the post-diversified amounts for these risks as well?
    - **A.** Yes**.** These are asked for on the Supplementary Questionnaire. Premium and reserve risk post-diversified amounts are very useful for Lloyd's review.
- **Q.** Why has the confidence level been set at 95%?
    - **A**. There must be a high degree of confidence that the "true" model SCR lies within the range used for the calculations.
- **Q.** Does the method have any relevance to stability testing/simulation error?
    - **A.** Yes. You can be 95% confident that the simulation error does not exceed the difference between the reported SCR and the upper/lower bound of the CI.
- **Q.** Are agents required to use the methodology in managing the business or to otherwise meet the Use test?
    - **A.** No.

## Appendix: Methodology for determining the ranges

Let

- $X$ be the random variable for the internal model balance sheet position
- n be the number of simulations
- $\pi_p$ be the (100p)th percentile of $X$
- $X_1, X_2, \ldots X_n$ be the n simulated balance sheet positions
- $Y_1 \leq Y_2 \leq \ldots \leq Y_n$ be the ordered (ranked) $X_k$

We also assume that the simulations are independent and constitute a random sample from the model.

The expected number of simulated $X_k$ less than or equal to the (100p)th percentile $\pi_p$ is np. The probability of observing $i$ simulations less than or equal to $\pi_p$ out of the total of n simulations is given by a binomial distribution with mean np and variance np(1-p).

$\Pr(\textbf{no. simulations} \leq \boldsymbol{\pi_p} = i) = n!/([i!]*[n-i]!)*p^i * [1-p]^{n-i}$

The probability of observing *at least* i simulations and *at most* j-1 simulations less than or equal to $\pi_p$ is

$\Pr(\, i \leq \textbf{no. simulations} \leq \boldsymbol{\pi_p} < j) = \Sigma\, n!/([k!]*[n-k]!)*p^k * [1-p]^{n-k}, \quad k = i, i+1, \ldots, j-1$ **(*)**

**(*)** can be approximated using the normal distribution:

$\Pr(\, i \leq \textbf{no. simulations} \leq \boldsymbol{\pi_p} < j) \cong \Phi([(j-1+0.5) - np]/[np*(1-p)]^{0.5}) - \Phi([(i-0.5) - np]/[np*(1-p)]^{0.5})$ **(**)**

(The "continuity correction" of +/-0.5 is made to improve the accuracy of the normal approximation.)

Let $j - 1 = np + \Delta$ and $i = np - \Delta$. We can rewrite **(**)** as

$\Pr(\, i \leq \textbf{no. simulations} \leq \boldsymbol{\pi_p} < j) \cong \Phi([(\Delta+0.5]/[np*(1-p)]^{0.5}) - \Phi(-[\Delta + 0.5]/[np*(1-p)]^{0.5})$

$= 2 * \Phi([(\Delta+0.5]/[np*(1-p)]^{0.5}) - 1$ **(+)**

We can use **(+)** to derive a confidence interval for $\pi_p$ that is symmetric around the (100p)th percentile in terms of the numbers of simulations.

- Select the desired confidence level $CL(\Delta)$
- Using **(+)**, set $CL(\Delta) = 2 * \Phi([(\Delta+0.5]/[np*(1-p)]^{0.5}) - 1$
- Solve for $\Delta = \Phi^{-1}([CL(\Delta)+1]/2) * [np*(1-p)]^{0.5} - 0.5$
- Calculate $j = np + \Delta + 1$ and $i = np - \Delta$ (round to the nearest integer)
- $[Y_i, Y_j)$ is the $CL(\Delta)$ confidence interval (CI) for $\pi_p$

The boundaries $Y_i$ and $Y_j$ follow from the definition of **(*)**, which gives the probability of *at least i* simulations and *at most j*-1 simulations less than or equal to $\pi_p$. Since the $Y_k$ are ordered, $Y_i$ and $Y_j$ are the smallest and largest simulations, respectively, consistent with our selected confidence level CL($\Delta$) for $\pi_p$ and the number of simulations n.

Remarks

- The application of the methodology to form 309 would assume the following.
  - p = 0.995
  - $\pi_p$ is the true internal model SCR
  - CL($\Delta$) is 95%;
  - $j = np + 1.96*[np(1-p)]^{0.5} + 0.5$
  - $i = np - 1.96*[np(1-p)]^{0.5} + 0.5$

- $(j-i)/n \propto ([1-p]p/n)^{0.5}$ i.e. the width of the interval relative to the number of simulations decreases with the square root of the number of simulations (law of large numbers)

- The method for determining the CI is non-parametric and therefore independent of the form (shape, mean, variance, etc.) of the distribution for *X*. The values for *i* and *j* will therefore define a CL($\Delta$) interval for the percentile p for any other ranked random variable in the internal model (assuming n simulations).

References


Hogg, R., J. McKean and A. Craig. *Introduction to Mathematical Statistics* (6th ed.), Upper Saddle River, NJ: Pearson Prentice Hall, 2005.

Hogg, R., and S. Klugman. *Loss Distributions*, USA: John Wiley & Sons, 1984.

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# THE SUM OF SQUARES TEST (SST)

**(Supplementary Questionnaire:** PartII_PremRiskExcCat_Outputs; PartII_PremRiskIncCat_Outputs; PartII_ResRisk_Outputs**)**

### Definitions

- X, Y and Z are random variables
- Z = X + Y
- $\rho$ is the correlation between X and Y

### Derivation of SST

MEAN(Z) = MEAN(X) + MEAN(Y)

VAR(Z) = VAR(X) + 2$\rho$STDEV(X)*STDEV(Y) + VAR(Y)

The above is true in general and does not depend on the distribution assumptions.

In the SST we set $\rho = 0$.

MEAN(Z) = MEAN(X) + MEAN(Y)

VAR(Z) = VAR(X) + VAR(Y), or

$\text{STDEV}(Z) = [\text{STDEV}(X)^2 + \text{STDEV}(Y)^2]^{1/2}$

In general, a given percentile p of a distribution will be equal to the mean plus some multiple $k_p$ of the standard deviation.

$X_p = \text{MEAN}(X) + k_p * \text{STDEV}(X)$ (*)

The value of $k_p$ will depend on the distribution and the percentile. For example, for the Normal at the 99.5th, $k_{99.5} = 2.57$; for the lognormal for many insurance risk distributions, $k_{99.5} \sim 3.0$. (The QIS5 technical specification uses this assumption; see SCR 9.17-9.18.)

Rearranging (*) gives

$\text{STDEV}(X) = [X_p - \text{MEAN}(X)]/ k_p$

If we **assume** that X, Y and Z all have the same distribution and (therefore $k_p$ ) then the $p^{th}$ percentile for Z is

$Z_p = [\text{MEAN}(X) + \text{MEAN}(Y)] + k_p * \{([X_p - \text{MEAN}(X)]/ k_p )^2 + ([Y_p - \text{MEAN}(Y)]/ k_p)^2\}^{1/2}$ (**)

The $k_p$ cancel out to give

$\mathbf{Z_p = [MEAN(X) + MEAN(Y)] + \{[X_p - MEAN(X)]^2 + [Y_p - MEAN(Y)]^2\}^{1/2}}$ (+)

The SST is applied by comparing the modelled result for $Z_p$ with the result from (+). The latter is taken as the result that would be obtained assuming independence between X and Y. For example, if X and Y are premium and reserve risk, then their means and 99.5ths can be obtained from form 314. If the $99.5^{th}$ for insurance risk is less than the result obtained from (+), then the SST is failed.

We can generalise (+) to more than two risks using matrix multiplication. The SST is applied in the Supplementary Questionnaire to the modelled classes of business for premium and reserve risk.

**Limitations**

The assumption supporting (**) above will be true if the distributions are normal. In this case the $k_p$ of X and Y will be the same; furthermore, since the sum of two normally distributed random variables has a normal distribution, Z will have the same $k_p$ as X and Y.

Conversely, the assumption supporting (**) will not be valid if:

- The distributions of X and Y are of different shape and their $k_p$ differ.
- The distributions of and X and Y are the same/similar but skewed.

In the second case, the sum of two random variables with the same non-normal distribution cannot be assumed to have that distribution. The more skewed the distributions, the less valid the assumption.

In the first case, the degree of mis-estimation by the SST will depend in part on the relative size of the standard deviations for X and Y in (**). If one is much larger than the other, then it will dominate the result in (+) and the impact of the differences in $k_p$ will be smaller, resulting in a smaller mis-estimation by the SST. This is the case in Scenario 2 below.

Due to the above limitations, Lloyd's considers the SST to be useful as a "first-pass" test only.

The table below compares the accuracy of the SST against simulated values under three different scenarios. All distributions are lognormal. The $k_{99.5}$ have been calculated analytically using (*) above.

| | **Scenario 1** Both X and Y low-skewed | | **Scenario 2** X skewed / Y low-skewed | | **Scenario 3** Both X and Y skewed | |
|---|---|---|---|---|---|---|
| | Risk X | Risk Y | Risk X | Risk Y | Risk X | Risk Y |
| Mean | 100 | 100 | 100 | 100 | 100 | 100 |
| COV | 10% | 10% | 50% | 10% | 50% | 50% |
| Skewness | 0.30 | 0.30 | 1.63 | 0.30 | 1.63 | 1.63 |
| $k_{99.5}$ | 2.87 | 2.87 | 4.04 | 2.87 | 4.04 | 4.04 |

The results obtained from the SST are compared with simulated results (n = 100,000) for the three scenarios. The error is negligible in Scenarios 1 and 2. (It would be higher in Scenario 2 if Y were larger.) The error is significant in Scenario 3 at higher percentiles. Note that simulation error is expected to increase from Scenarios 1 -3.

| **Risk X + Risk Y: SST vs. Simulations** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **Scenario 1** | | | **Scenario 2** | | | **Scenario 3** | | |
| Percentile | SST | Sim | %error | SST | Sim | %error | SST | Sim | %error |
| 50.0% | 200.64 | 199.58 | 0.53% | 210.26 | 190.22 | 9.53% | 214.66 | 188.89 | 12.01% |
| 75.0% | 209.18 | 209.34 | -0.08% | 224.55 | 224.68 | -0.06% | 233.03 | 237.64 | -1.98% |
| 90.0% | 218.53 | 218.53 | 0.00% | 265.65 | 265.62 | 0.01% | 290.59 | 292.92 | -0.80% |
| 95.0% | 224.42 | 224.09 | 0.15% | 297.07 | 296.70 | 0.12% | 333.91 | 332.05 | 0.56% |
| 97.5% | 229.68 | 229.25 | 0.19% | 328.29 | 327.85 | 0.13% | 377.80 | 369.98 | 2.07% |
| 99.0% | 235.96 | 234.96 | 0.43% | 371.43 | 371.22 | 0.06% | 437.27 | 419.42 | 4.08% |
| 99.5% | 240.44 | 238.86 | 0.66% | 405.61 | 404.01 | 0.39% | 484.74 | 453.46 | 6.45% |
| 99.8% | 245.64 | 244.20 | 0.59% | 449.09 | 446.65 | 0.54% | 545.60 | 510.79 | 6.38% |