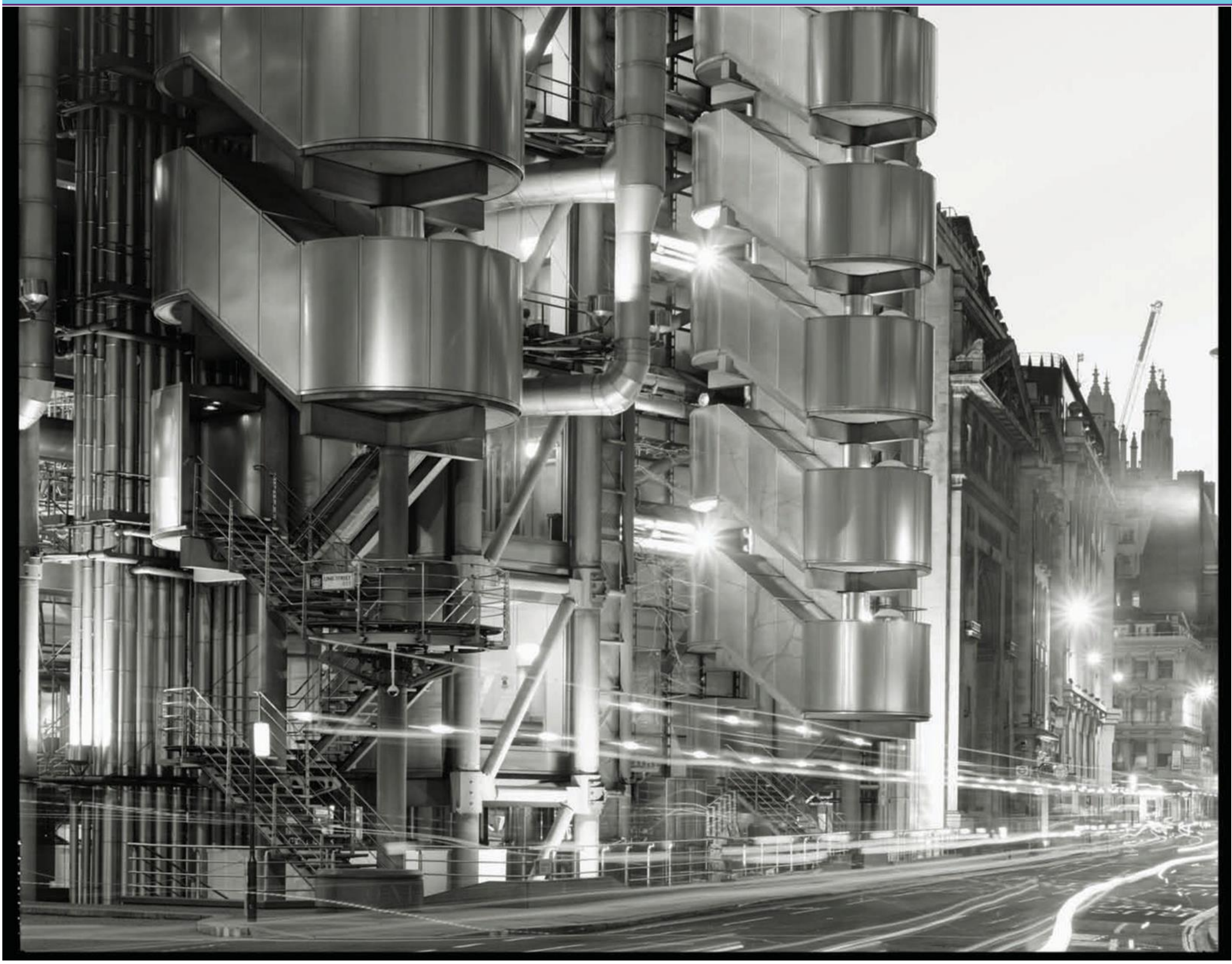
SYNDICATE SCR FOR 2017 YEAR OF ACCOUNT

NOTES TO THE SUPPLEMENTARY QUESTIONNAIRE AND ANALYSIS OF CHANGE

JULY 2016



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PURPOSE

This document provides additional information on certain calculations and tests used in the Supplementary Questionnaire and Analysis of Change template, for the SCR submissions at Lloyd's. It is a supporting document only and contains no additional requirements or guidance.

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The YOA 2017 SCR Supplementary Questionnaire is required by 1pm 13 September 2016.

The YOA 2017 SCR Analysis of Change is required by 1pm 16 September 2016.

SUMMARY OF CHANGES IN THE SUPPLEMENTARY QUESTIONNAIRE

Part I – Qualitative

All the questions from last year are still included. Further questions have been added to the following sections:

- "PartI_PremRisk" now includes questions on the modelling of multi-year policies, the widening of terms and conditions and exposure to cyber risk.
- "PartI_SCR" now includes a question regarding whether the reinsurance contract boundaries change in the Technical Provisions guidance has been included in the model (see Part II below).

Lloyd's review in 2016, as mentioned in the SCR Guidance, will focus on modelling of multi-year policies and its impact on the capital, uncertainty in loss distribution due to widening of terms and conditions and the impact on the parameters, and finally information on the parameterisation and validation of cyber risk. Impact of silent cyber will also be reviewed. Depending on the answer to the first question and information provided in the documents submitted along with the LCR, Lloyd's might request further information.

Part II – Quantitative

In addition to the information requested last year, a section to reconcile key inputs to the SBF (Syndicate Business Forecast) and LCR (Lloyd's Capital Return) has been added.

SBF vs. LCR reconciliation

This section includes reconciliation between different SBF and LCR inputs. This reconciliation was previously included in the SCR feedback from Lloyd's.

The first question compares mean net claims and net premium. The mean values are expected to be consistent between the business plan and the internal model. A brief explanation should be provided for any difference in values.

The mean profit in the SBF and LCR can be different for a few reasons. The reasons for the difference should be included to complete the reconciliation. If the reason for difference (e.g. profit from prior years) is not listed in the table, please add it under "other" and complete the reconciliation. This exercise will assist Lloyd's in the SCR reviews and for LIM validation.

Part III – Reinsurance contract boundaries

As for the 2016 year of account, an adjustment will be made to remove the impact of the change to consideration of reinsurance contract boundaries. The change and nature of the adjustment is described in the latest SCR guidance and detailed examples on the impact released last year.

This section derives the SCR adjustment to be entered in the LCR form 309, table 1, row 2 of the LCR. Last year the impact was collected in a standalone template – SCR Contract Boundaries Change Return, this section of the supplementary questionnaire replaces the standalone contract boundaries template.

As outlined in the Supplementary Questionnaire the TPs used for the adjustment should be those underlying the Q2 QMC submission and the opening model position as at Q4.

Syndicates are required to enter the RI contract boundary adjustment (calculated in worksheet 'PartIII_RICB_Outputs', cell F35) into the LCR form 309, Table 1, Row 2. This is mandatory for the year-end CiL submission (September) and should only be completed for the mid-year CiL (March) in the event of resubmission of the LCR. Lloyd's is expecting to use information from the Q4 QMC in all other cases.

STANDARDISED CALCULATION OF THE POST-DIVERSIFICATION AMOUNTS ON FORM 309

(Supplementary Questionnaire: PartII_PostDiv_Inputs**)**

Purpose

Beginning with the 2015 YOA LCR submission, agents have been required to use a standardised methodology for calculating the post-diversification values shown on form 309. This note provides a step-by-step illustration of how to apply the method. It does not contain any guidance or requirements that are not specified in the SCR Guidance or the Supplementary Questionnaire and is for informational purposes only.

Executive Summary

Form 309 provides columns for the post-diversified amounts of each SCR risk category. These post-diversified amounts are intended to represent the contribution of each risk category to the SCR.

In summary, in order to dervice these amounts on both a one-year and ultimate basis the following process should be used:

(i) Rank the simulated balance sheet positions (the SCR is the $VaR_{99.5}$ of the balance sheet position)

(ii) Calculate a proxy "Confidence Interval SCR" (CI SCR) by averaging over a range of simulations specified by Lloyd's

(iii) Calculate the average amounts for each risk category over the range of simulations used in (ii).

(iv) Calculate the post-diversified amounts for each risk category by scaling the averages by the ratio of the syndicate's selected post-diversified SCR to the proxy CI SCR

(v) Report the post-diversified amounts on form 309 columns E and I

The above methodology applies only to the calculation of the post-diversified SCR risk categories in rows 1-8 of columns E and I of form 309 **excluding** the SCR (row 9). The diversified SCR is to be calculated according to a method judged appropriate by the agent, consistent with the SCR Guidance 4.11 - 4.12. The "proxy CI SCR" is an intermediate value to be discarded after being used to determine the post-diversified amounts for the SCR risk categories.

The range of simulations are defined to ensure that the "true" internal model 99.5^{th} percentile SCR (i.e. the value that the SCR converges to as simulation error approaches nil) lies within the range at a 95% confidence level. Details are in the Appendix.

The approach is based on outputs currently produced for the LCR. There should be no need for agents to revise their model structure or calculate new metrics.

What agents must do: Simple numerical example

The purpose of the following example is to illustrate the methodology; it is not intended to provide a realistic example of syndicate model outputs or the number of simulations required. The example is shown for the ultimate case (column I on form 309); the same methodology applies to the one-year case (column E on form 309).

Suppose the model was run for 10,000 simulations and the 1:200 pre-diversification amounts by risk category are as shown below. The ultimate SCR is 122.4m.

				Ultimate basis (Note 309.2)	
				Pre diversification	Post diversification
				GBP (m)	GBP (m)
				E	G
Insurance Risk	total:	After diversification between Premium and Reserve Risk	1	112.5	
	split:	Premium Risk (see note above)	2	80.9	
	split:	Reserve Risk (Note 309.4)	3	48.8	
Credit Risk	total:	After diversification between Reinsurance and Other Credit Risk	4	18.1	
	split:	Reinsurance Credit Risk	5	17.2	
	split:	Other Credit Risk	6	2.6	
Market Risk (see note above)			7	22.9	
Operational Risk			8	15.0	
TOTAL (Note 309.3)			9	168.5	
Diversification Credit - between risk categories			10	- 46.1	
DIVERSIFIED TOTAL (Note 309.3)			11	**122.4**	

The steps to calculate the post-diversification amounts to be shown in column I are as follows. (The corresponding section in the Supplementary Questionnaire is referred to in brackets.)

1. **Rank the simulated balance sheet positions from smallest to largest.** (SuppQ PartII_PostDiv_Inputs Q1)

Simulation No.	Risk Type: Premium	Reserve	Insurance	RI credit	Other credit	Credit	Market	Operational	Balance sheet position	Balance sheet position rank
3526	-61.4	-93.5	-154.8	0.1	0.0	0.1	-22.1	2.0	-174.9	1
239	-63.9	-80.5	-144.4	0.0	0.1	0.1	-16.8	0.0	-161.2	2
7637	-56.4	-94.3	-150.7	10.0	0.1	10.1	-18.1	2.0	-156.8	3
6284	87.3	109.8	197.2	10.0	0.0	10.0	1.3	0.0	208.4	9998
5101	169.5	26.2	195.7	10.0	0.0	10.0	12.4	0.0	218.1	9999
6584	166.3	63.7	230.0	0.0	0.1	0.1	14.0	0.0	244.0	10000

Simulations sorted based on size of simulated balance sheet position

2. **Determine the appropriate range of simulations for the post-diversification calculations from ranges provided by Lloyd's.** (SuppQ PartII_PostDiv_Inputs Q1) The simulation ranges have been selected to provide a 95% confidence interval for the "true" internal model SCR. See the Appendix for details on the methodology.

   Since 10,000 simulations have been run, the range for the post-diversification calculations would be from 9,937 to 9,964 *after* sorting by ascending size of the balance sheet position.

Post-diversification calculations: specification of ranges							
No. simulations	**10,000**	25,000	50,000	75,000	150,000	200,000	250,000
SCR percentile	**99.5**	99.5	99.5	99.5	99.5	99.5	99.5
Confidence level that SCR percentile lies in range	**95.00%**	95.00%	95.00%	95.00%	95.00%	95.00%	95.00%
Range definition in terms of rank of SCR simulations							
upper bound	**9,964**	24,897	49,781	74,663	149,304	199,062	248,819
lower bound	**9,937**	24,854	49,720	74,588	149,197	198,939	248,682
range width	**28**	44	62	76	107	124	137

3. **Determine the proxy CI SCR and average values for each SCR risk type over the specified range of simulations.** (SuppQ PartII_PostDiv_Inputs Q2 & Q3 and PartII_PostDiv_Outputs M1)

   The specified range from simulation 9,937 to 9,964 is shown below**.**

Simulation No.	Risk Type Premium	Reserve	Insurance	RI credit	Other credit	Credit	Market	Operational	Balance sheet position	Balance sheet position rank
3526	-61.4	-93.5	-154.8	0.1	0.0	0.1	-22.1	2.0	-174.9	1
239	-63.9	-80.5	-144.4	0.0	0.1	0.1	-16.8	0.0	-161.2	2
.....										
5242	81.2	29.9	111.1	0.0	0.5	0.5	-9.2	0.0	114.7	9937
8306	69.9	44.4	114.3	10.0	0.0	10.0	-9.3	0.0	115.0	9938
.....										
1413	43.6	71.6	115.2	0.0	0.0	0.0	-2.8	15.0	127.4	9963
736	17.4	90.7	108.1	0.1	0.1	0.1	12.5	7.5	128.2	9964
5211	1.7	121.9	123.6	0.1	0.5	0.6	-3.3	7.5	128.4	9965
.....										
5101	169.5	26.2	195.7	10.0	0.0	10.0	12.4	0.0	218.1	9999
6584	166.3	63.7	230.0	0.0	0.1	0.1	14.0	0.0	244.0	10000

Balance sheet position and risk types averaged over these simulations

For example, insurance risk would be averaged over the values 111.1, 114.3,…, 115.2, 108.1 = 116.0. This is just insurance risk averaged over the 28 simulations for which the rank of the balance sheet position falls within the defined range.

The proxy CI SCR would be the average of 114.7, 115.0, …,127.4, 128.2 = 122.8.

The post-diversified insurance risk would be 116.0 * (selected SCR)/(proxy CI SCR) = 116.0 * (122.4/122.8) = 115.6. This is the amount that would be shown as post-diversified insurance risk on form 309. The results for the other risks are shown below. The scaling factor ensures that their sum is equal to the selected diversified SCR of 122.4m shown in row 11 of column G of form 309.

Form 309 reported SCR (m):	122.4		
SCR Risk Type	CI Value (m)	Scaling Factor	Post Diversified (m) (for Form 309 col G)
Insurance	116.0	99.6%	115.6
Premium	74.0		73.7
Reserve	42.0		41.9
Credit	2.5		2.5
RI credit	1.9		1.9
Other credit	0.7		0.7
Market	0.9		0.9
Operational	3.4		3.3
CI SCR	122.8		122.4

4. Populate rows 1-8 of col I of from 309 with the post-diversified amounts from step 3. Discard the CI SCR. (PartII_PostDiv_Outputs M1)

				Ultimate basis (Note 309.2)	
				Pre diversification	Post diversification
				GBP (m)	GBP (m)
				E	G
Insurance Risk	total:	After diversification between Premium and Reserve Risk	1	112.5	115.6
	split:	Premium Risk (see note above)	2	80.9	
	split:	Reserve Risk (Note 309.4)	3	48.8	
Credit Risk	total:	After diversification between Reinsurance and Other Credit Risk	4	18.1	2.5
	split:	Reinsurance Credit Risk	5	17.2	
	split:	Other Credit Risk	6	2.6	
Market Risk (see note above)			7	22.9	0.9
Operational Risk			8	15.0	3.3
TOTAL (Note 309.3)			9	168.5	
Diversification Credit - between risk categories			10	- 46.1	
DIVERSIFIED TOTAL (Note 309.3)			11	**122.4**	

The post-diversified values for reserve risk, premium risk, RI credit risk and other credit risk are reported on the Supplementary Questionnaire.

Q&A

- **Q**. Why average over the range instead of taking individual values corresponding to the simulated 99.5^{th} percentile balance sheet position?
    - **A.** Model outputs for the SCR risk types can vary considerably by simulation around a given percentile. The average will be more stable.
- **Q**. Why bother with the proxy CI SCR? Why not just use the selected diversified SCR?
    - **A.** The post-diversified SCR risk categories should sum to the selected diversified SCR. This will not normally be the case, unless the selected SCR is exactly equal to the proxy CI SCR. The CI SCR is therefore needed to scale the averaged SCR risk category amounts. In most cases the scaling adjustment should be small.
- **Q**. Is this approach required for other 1:200 values on the LCR, e.g. the pre-diversified amounts on form 309 or the claim amounts on form 313?
    - **A.** No, there are currently no plans to require the approach to be applied to other parts of the LCR.
- **Q.** The LCR does not show the post-diversified amounts for premium risk and reserve risk, only for insurance risk in total. Are agents required to calculate the post-diversified amounts for these risks as well?
    - **A.** Yes**.** These are asked for on the Supplementary Questionnaire. Premium and reserve risk post-diversified amounts are very useful for Lloyd's review.
- **Q.** Why has the confidence level been set at 95%?
    - **A**. There must be a high degree of confidence that the "true" model SCR lies within the range used for the calculations.
- **Q.** Does the method have any relevance to stability testing/simulation error?
    - **A.** Yes. You can be 95% confident that the simulation error does not exceed the difference between the reported SCR and the upper/lower bound of the CI.
- **Q.** Are agents required to use the methodology in managing the business or to otherwise meet the Use test?
    - **A.** No.

Appendix: Methodology for determining the ranges

Let

- X be the random variable for the internal model balance sheet position
- n be the number of simulations
- π_p be the (100p)th percentile of X
- $X_1, X_2, \ldots X_n$ be the n simulated balance sheet positions
- $Y_1 \leq Y_2 \leq \ldots \leq Y_n$ be the ordered (ranked) X_k

We also assume that the simulations are independent and constitute a random sample from the model.

The expected number of simulated X_k less than or equal to the (100p)th percentile π_p is np. The probability of observing i simulations less than or equal to π_p out of the total of n simulations is given by a binomial distribution with mean np and variance np(1-p).

$\Pr(\textbf{no. simulations} \leq \boldsymbol{\pi_p} = i) = n!/([i!]*[n-i]!)*p^i * [1-p]^{n-i}$

The probability of observing *at least* i simulations and *at most* j-1 simulations less than or equal to π_p is

$\Pr(i \leq \textbf{no. simulations} \leq \boldsymbol{\pi_p} < j) = \Sigma\, n!/([k!]*[n-k]!)*p^k * [1-p]^{n-k}, \quad k = i, i+1, \ldots, j-1$ **(*)**

(*) can be approximated using the normal distribution:

$\Pr(i \leq \textbf{no. simulations} \leq \boldsymbol{\pi_p} < j) \cong \Phi([(j-1+0.5) - np]/[np*(1-p)]^{0.5}) - \Phi([(i-0.5) - np]/[np*(1-p)]^{0.5})$ **(**)**

(The "continuity correction" of +/-0.5 is made to improve the accuracy of the normal approximation.)

Let $j - 1 = np + \Delta$ and $i = np - \Delta$. We can rewrite **(**)** as

$\Pr(i \leq \textbf{no. simulations} \leq \boldsymbol{\pi_p} < j) \cong \Phi([(\Delta+0.5]/[np*(1-p)]^{0.5}) - \Phi(-[\Delta + 0.5]/[np*(1-p)]^{0.5})$

$= 2 * \Phi([(\Delta+0.5]/[np*(1-p)]^{0.5}) - 1$ **(+)**

We can use **(+)** to derive a confidence interval for π_p that is symmetric around the (100p)th percentile in terms of the numbers of simulations.

- Select the desired confidence level CL(Δ)
- Using **(+)**, set $CL(\Delta) = 2 * \Phi([(\Delta+0.5]/[np*(1-p)]^{0.5}) - 1$
- Solve for $\Delta = \Phi^{-1}([CL(\Delta)+1]/2) * [np*(1-p)]^{0.5} - 0.5$
- Calculate $j = np + \Delta + 1$ and $i = np - \Delta$ (round to the nearest integer)
- $[Y_i, Y_j)$ is the CL(Δ) confidence interval (CI) for π_p

The boundaries Y_i and Y_j follow from the definition of **(*)**, which gives the probability of *at least i* simulations and *at most j*-1 simulations less than or equal to π_p. Since the Y_k are ordered, Y_i and Y_j are the smallest and largest simulations, respectively, consistent with our selected confidence level CL(Δ) for π_p and the number of simulations n.

Remarks

- The application of the methodology to form 309 would assume the following.
    - p = 0.995
    - π_p is the true internal model SCR
    - CL(Δ) is 95%;
    - $j = np + 1.96*[np(1-p)]^{0.5} + 0.5$
    - $i = np – 1.96*[np(1-p)]^{0.5} + 0.5$

- $(j-i)/n \propto ([1-p]p/n)^{0.5}$ i.e. the width of the interval relative to the number of simulations decreases with the square root of the number of simulations (law of large numbers)

- The method for determining the CI is non-parametric and therefore independent of the form (shape, mean, variance, etc.) of the distribution for *X*. The values for *i* and *j* will therefore define a CL(Δ) interval for the percentile p for any other ranked random variable in the internal model (assuming n simulations).

References

Hogg, R., J. McKean and A. Craig. *Introduction to Mathematical Statistics* (6th ed.), Upper Saddle River, NJ: Pearson Prentice Hall, 2005.

Hogg, R., and S. Klugman. *Loss Distributions*, USA: John Wiley & Sons, 1984.

THE SUM OF SQUARES TEST (SST)

(Supplementary Questionnaire: PartII_PremRiskExcCat_Outputs; PartII_PremRiskIncCat_Outputs; PartII_ResRisk_Outputs**)**

Definitions

- X, Y and Z are random variables
- Z = X + Y
- ρ is the correlation between X and Y

Derivation of SST

MEAN(Z) = MEAN(X) + MEAN(Y)

VAR(Z) = VAR(X) + 2ρSTDEV(X)*STDEV(Y) + VAR(Y)

The above is true in general and does not depend on the distribution assumptions (provided the moments exist).

In the SST we set $\rho = 0$.

MEAN(Z) = MEAN(X) + MEAN(Y)

VAR(Z) = VAR(X) + VAR(Y), or

$\text{STDEV}(Z) = [\text{STDEV}(X)^2 + \text{STDEV}(Y)^2]^{1/2}$

In general, a given percentile p of a distribution will be equal to the mean plus some multiple k_p of the standard deviation.

$X_p = \text{MEAN}(X) + k_p * \text{STDEV}(X)$ (*)

The value of k_p will depend on the distribution and the percentile. For example, for the Normal at the 99.5th, $k_{99.5} = 2.57$; for the lognormal for many insurance risk distributions, $k_{99.5} \sim 3.0$. (The QIS5 technical specification uses this assumption; see SCR 9.17-9.18.) The value of k_p will be negative for percentiles below the mean. In the following derivation, we will assume that the p^{th} percentile lies above the mean.

Rearranging (*) gives

$\text{STDEV}(X) = [X_p - \text{MEAN}(X)]/ k_p$

If we **assume** that X, Y and Z all have the same distribution and (therefore k_p) then the p^{th} percentile for Z is

$Z_p = [\text{MEAN}(X) + \text{MEAN}(Y)] + k_p * \{([X_p - \text{MEAN}(X)]/ k_p)^2 + ([Y_p - \text{MEAN}(Y)]/ k_p)^2\}^{1/2}$ (**)

The k_p cancel out to give

$\mathbf{Z_p = [\text{MEAN}(X) + \text{MEAN}(Y)] + \{[X_p - \text{MEAN}(X)]^2 + [Y_p - \text{MEAN}(Y)]^2\}^{1/2}}$ (+)

The SST is applied by comparing the modelled result for Z_p with the result from (+). The latter is taken as the result that would be obtained assuming independence between X and Y. For example, if X and Y are premium and reserve risk, then their means and 99.5ths can be obtained from form 314. If the 99.5^{th} for insurance risk is less than the result obtained from (+), then the SST is failed.

We can generalise (+) to more than two risks using matrix multiplication. The SST is applied in the Supplementary Questionnaire to the modelled classes of business for premium and reserve risk.

Advantages

The SST is a simple calculation that requires only the means and percentiles p of the marginal distributions. It is the only way to approximate the sum of two random variables without relying on simulation or the fast Fourier transform.

Limitations

The assumption supporting (**) above will be true if the distributions are normal. In this case the k_p of X and Y will be the same; furthermore, since the sum of two normally distributed random variables has a normal distribution, Z will have the same k_p as X and Y.

Conversely, the assumption supporting (**) will not be valid if:

- The distributions of X and Y are of different shape and their k_p's differ.
- The distributions of X and Y are the same/similar but skewed.

In the second case, the sum of two random variables with the same non-normal distribution cannot be assumed to have that distribution. The more skewed the distributions, the less valid the assumption. In addition, there is a greater risk that the mean for one of the distributions lies above the percentile at which the SST is applied. This will increase the error in (+).

In the first case, the degree of mis-estimation by the SST will depend in part on the relative size of the standard deviations for X and Y in (**). If one is much larger than the other, then it will dominate the result in (+) and the impact of the differences in k_p will be smaller, resulting in a smaller mis-estimation by the SST.

In summary, the limitations of the SST arise from approximating distributions using only the first two moments.

The advantages and limitations of the SST can be demonstrated with simulated outputs. Low-skewed risks were simulated from lognormals with means of 100 and standard deviations of 10; skewed risks were simulated from a frequency distribution with probability of a claim equal to 1/100 and a beta severity distribution with mean 10,000. The means of the low- and high-skewed distributions were therefore the same (100), but the standard deviations and skewness differed significantly. The lognormal resembles a distribution that could be appropriate for attritional claims or aggregate reserves; the discrete frequency/beta is more like the frequency/severity distribution for a nat cat portfolio. The low- and high-skewed distributions were simulated (n = 100,000) from independently in three different pairings; the results for the marginal risk distributions are shown below.

	Scenario 1 Both X and Y low-skewed		**Scenario 2** X skewed / Y low-skewed		**Scenario 3** Both X and Y skewed	
	Risk X	Risk Y	Risk X	Risk Y	Risk X	Risk Y
Mean	100.0	100.0	96.1	100.0	99.4	99.1
Std Dev	10.0	10.0	1089.8	10.0	1116.3	1068.4
COV	10%	10%	1135%	10%	1123%	1135%
Skewness	0.30	0.31	12.95	0.30	12.8	13.0
$k_{99.5}$	2.9	2.8	7.3	2.9	7.6	8.0

The second table shows the results for the aggregate of Risk X and Risk Y obtained from the SST compared to the simulated results in each of the three scenarios. Scenario 1 combines two low-skewed distributions; the error is negligible as expected. In Scenario 2, the %error is very high until above the 99th percentile. This is because the (unadjusted) SST implicitly assumes that these percentiles are above the mean; however, for the skewed risk, the expected probability of a claim is 1/100, and the mean occurs above the 99th percentile. The error is negligible above the 99th percentile because the skewed distribution completely dominates the low-skewed distribution. In Scenario 3, the error is high until above the 99th, for the same reason as in Scenario 1. The error remains significant at the 99.8th percentile, primarily because the high skewness increases simulation error.

Risk X + Risk Y: SST vs. Simulations									
	Scenario 1			Scenario 2			Scenario 3		
%ile	SST	Sim	%error	SST	Sim	%error	SST	Sim	%error
50.0%	200.7	199.5	0.6%	292.1	99.7	193.1%	330.4	0.0	N/A
75.0%	209.1	209.3	-0.1%	292.3	106.7	174.1%	330.4	0.0	N/A
90.0%	218.5	218.4	0.0%	293.0	113.7	157.8%	330.4	0.0	N/A
95.0%	224.4	224.2	0.1%	293.7	118.5	147.9%	330.4	0.0	N/A
97.5%	229.7	229.2	0.2%	294.4	123.5	138.4%	330.4	0.0	N/A
99.0%	236.1	235.2	0.4%	295.5	140.3	110.6%	330.4	9206.0	-96.4%
99.5%	240.5	239.0	0.6%	9270.5	9276.5	-0.1%	13105.3	13447.7	-2.5%
99.8%	246.2	244.4	0.7%	14662.4	14649.3	0.1%	20638.9	17144.6	20.4%

Use in the Supplementary Questionnaire

The SST is applied in the Supplementary Questionnaire to the class of business distributions for premium risk and reserve risk. The limitations described above imply that it will often be less appropriate for premium risk, for which the class distributions are frequently skewed (particularly for catastrophe classes) and differently shaped from each other. It is more likely to be reliable for reserve risk, for which the class distributions tend to have means nearer the median. Lloyd's uses the SST as a "first pass" test only. The SST is given less weight at lower percentiles, due to the errors that may be introduced if the mean corresponds to a higher percentile. It is a test of diversification rather than dependency, since the result is determined by both the marginal distributions and the dependencies between them. In cases where the SST indicates near independence, other metrics or model outputs may be requested, such as joint exceedance probabilities.

JOINT EXCEEDANCE PROBABILITIES

(Supplementary Questionnaire: PartII_PremRiskExcCat_Outputs; PartII_PremRiskIncCat_Outputs; PartII_ResRisk_Outputs; PartII_Dependencies_Outputs**)**

Definitions

Given random variables X and Y with marginal distributions F(x) and F(y) and joint distribution F(x,y), the probability that X>x and Y>y is given by the survival or tail function S(x,y):

$S(x,y) = 1 – F(x) – F(y) + F(x,y)$.

If F(x) and F(y) are chosen to equal a quantile or percentile p, then S(x,y) is the joint quantile exceedance probability or "JEP" at p.

The upper and lower bounds for F(x,y) are given by

$F_u(x,y) = \min[F(x), F(y)]$

$F_l(x,y) = \max[F(x) + F(y) – 1, 0]$

These correspond to full dependence (comonotonicity) and "negative" dependence (countermonotonicity), respectively. They are often referred to as the Frechet upper and lower bounds.

Similarly, the joint distribution corresponding to independent X and Y is

$F_i(x,y) = F(x)*F(y)$.

In general, Lloyd's does not consider levels of dependency below independence to be acceptable for LCR risks. The lower bound shown in the Supplementary Questionnaire graphs corresponds to independent marginals:

$S_i(x,y) = 1 – F(x) – F(y) + F_i(x,y) = 1 – F(x) – F(y) + F(x)*F(y)$.

The upper bound corresponding to full dependence is given by

$S_u(x,y) = 1 – F(x) – F(y) + F_u(x,y) = 1 – F(x) – F(y) + \min[F(x), F(y)]$.

For example, at F(x) = F(y) = 0.900,

$S_i(x,y) = 1 – 0.900 – 0.900 + 0.900^2 = 0.010$ and

$S_u(x,y) = 1 – 0.900 – 0.900 + \min[0.900, 0.900] = 0.100$.

The values for $S_i(x,y)$ and $S_u(x,y)$ can also be derived using the rule of conditional probability:

Pr(Y>y, X>x) = Pr(Y>y| X>x)*Pr(X>x), with Pr(Y>y| X>x) = 1 for full dependence and Pr(Y>y | X>x) = Pr(Y>y) for independence.

Optional inputs: Clayton and Gumbel copulae

The Supplementary Questionnaire allows for optional comparison of the modelled JEP with two copulae, the Clayton and the Gumbel. The Clayton has a nil coefficient of upper tail dependence while the Gumbel's is positive. The required input is Kendall's tau rank correlation coefficient. Note that the coefficient of upper tail dependence is defined in the limit as the percentile $p \to 1^-$; if the coefficient is nil, this does not imply independence at percentiles below 1.

The Clayton copula has a closed form given by

$C^{Cl}_{\alpha}(p_1,p_2) = (p_1^{-\alpha} + p_2^{-\alpha})^{-1/\alpha}$.

The Kendall's tau rank correlation coefficient for the Clayton is given by $\tau = \alpha / (\alpha + 2)$. Given τ, the joint exceedance probabilities can be calculated using the survival copula (corresponding to the definition of the survival function S(x,y) above):

$\underline{C}^{Cl}_{\alpha}(p_1,p_2) = 1 - p_1 - p_2 + C^{Cl}_{\alpha}(p_1,p_2) = 1 - p_1 - p_2 + (p_1^{-\alpha} + p_2^{-\alpha})^{-1/\alpha}$.

Similarly for the Gumbel:

$C^{Gu}_{\alpha}(p_1,p_2) = \exp\{-((-LN\ p_1)^{-\alpha} + (-LN\ p_2)^{-\alpha})^{-1/\alpha}\}$

$\underline{C}^{Gu}_{\alpha}(p_1,p_2) = 1 - p_1 - p_2 + \exp\{-((-LN\ p_1)^{-\alpha} + (-LN\ p_2)^{-\alpha})^{-1/\alpha}\}$

The Kendall's tau rank correlation coefficient is given by $\tau = 1 - 1/\alpha$.

The option to display the Clayton and Gumbel allows the model JEP to be compared in terms of strength and shape to upper tail independent and dependent copulae, both of which are defined by the same parameter τ.

References

McNeil, A., R. Frey and P. Embrechts. *Quantitative Risk Management*, Princeton, NJ: Princeton University Press, 2005.

Wang, S.S., "Aggregation of Correlated Risk Portfolios: Models and Algorithms", PCAS LXXXV, 1998, pp. 848 - 939

RI CREDIT RISK

(Supplementary Questionnaire: PartII_RI_Inputs and PartII_RI_Outputs**)**

The RI credit risk inputs require recoveries from defaulting reinsurers both gross and net of the credit risk loss or bad debt.

To illustrate the inputs needed to complete this section of the Questionnaire, we will assume that the syndicate has only one RI programme for the proposed YOA and that this applies to unbound new business. We ignore any reinsurance applying to the claims or premium provisions and assume that there is no bad debt provision.

Limit (£)	8,000,000
Excess (£)	2,000,000

Also assume that there are two reinsurers on the programme, and that their probabilities of default are independent.

Reinsurer	Probability of Default	Loss Given Default	Line	Exposure (£)
X	5%	50%	25%	2,000,000
Y	10%	50%	50%	4,000,000

We will make the simplifying assumption that the only possible gross loss is a single total loss to the layer. There are four possible outcomes for reinsurer default (the RI credit risk loss), given that a recovery is owed. The probability of each outcome is Pr(Default amount | Claim amount); the cumulative probabilities are shown below.

	Cumulative probability of RI credit risk loss given that a claim occurs (£)				
RI Scenario		Neither X nor Y defaults	X defaults but not Y	Y defaults but not X	X and Y default
Distribution value/ Amount	Mean	85.5%	90.0%	99.5%	100.0%
Reinsurer X	50,000	0	1,000,000	0	1,000,000
Reinsurer Y	200,000	0	0	2,000,000	2,000,000
Total	**250,000**	**0**	**1,000,000**	**2,000,000**	**3,000,000**

For example, the probability of Reinsurer X but not Reinsurer Y defaulting is 0.05 * (1 – 0.10) = 4.5%. This occurs at the 90th percentile since the probability of no default is (1 – 0.05)*(1 – 0.10) = 85.5%. Since the loss is a total loss, the recovery owed by Reinsurer X is £2.0m and the size of the default is £2.0m * 50% = £1.0m.

In order to obtain the unconditional RI credit risk distribution, we will assume that the probability of a claim is 90%. The probability of default is Pr(Default amount | Claim amount) * Pr(Claim amount). The full RI credit risk distribution is the following.

		RI credit risk distribution value (£)				
	Mean	10.00%	86.95%	91.00%	99.55%	100.00%
Gross Claim	9,000,000	0	10,000,000	10,000,000	10,000,000	10,000,000
RI scenario		No claim; no default	Neither X nor Y defaults	X defaults but not Y	Y defaults but not X	X and Y default
RI credit risk loss on RI recovery	225,000	0	0	1,000,000	2,000,000	3,000,000
RI recovery (gross) - all counterparties	5,400,000	0	6,000,000	6,000,000	6,000,000	6,000,000
RI recovery (gross) - Defaulting counterparties only	450,000	0	0	2,000,000	4,000,000	6,000,000
RI credit risk loss vs. RI recovery (Gross) - all counterparties	28.7%	-	-	16.7%	33.3%	50.0%
RI credit risk loss vs. RI recovery (Gross) - defaulting counterparties only	50.0%	-	-	50.0%	50.0%	50.0%

For example, the outcome of X but not Y defaulting occurs at the 91st percentile: 100%*0.1 + 85.5%*0.9 + 4.5%*0.9 = 91%.

The last two rows are produced in PartII_RI_Outputs. The final row quantifies the loss given default. Lloyd's has advised the market that this should be no less than 50%, before consideration of any mitigating factors such as collateral.

A key point is that the "RI recovery (gross) -Defaulting counterparties" captures only recoveries owed from the defaulting reinsurers – not the full gross recoveries owed on the programme.

BINARY EVENTS: CREDIT TO THE ULTIMATE SCR

(Supplementary Questionnaire: PartII_Dependencies_Inputs**)**

Background

The SCR Guidance states the following:

> *5.13 The balance sheet at T0 includes allowances for certain administrative and investment expenses and binary events that increase TPs. These may be more extreme than 1:200 so that the ultimate SCR effectively credits them back when considering the aggregate cash flows at the required stress point. This credit is produced implicitly in the ultimate SCR calculation; no offset to reserve risk or premium risk is required, in contrast to the credit from the risk margin. A worked example will be included in the updated "SCR Supplementary Questionnaire – Notes" available on lloyds.com. The size of the reduction must be stated in the supporting SCR methodology document and is also required as an input in the Supplementary Questionnaire.*

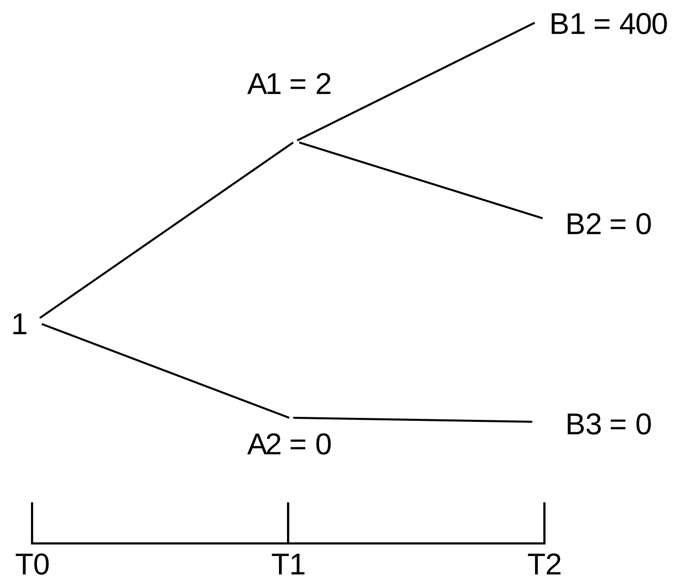
The 1:200 referred to is that of the balance sheet distribution, not the binary events distribution. (The 1:200 of the balance sheet distribution is the SCR, as shown on form 310.)

The PartII_Dependencies_Inputs Q3 of the Supplementary Questionnaire requires agents to report the credit to premium and reserve risk from binary events more extreme than 1:200. These amounts are captured for informational purposes; they are not used in any test.

The following simple numerical example demonstrates how the credit is produced. The format used is the same as that used for the premium risk examples in the SCR Guidance Numerical Examples. Refer to this document for an explanation of the tables below.

Example

Suppose the technical provisions include allowance for a binary event of £400m with a 1-in-400 return period. The supporting asset on the T0 balance sheet is £1m, plus the associated risk margin. The tree diagram below shows the distribution of ultimates (estimated at T0) at T1 and T2 (ultimate).



For this example, we will ignore the syndicate's other liabilities and the risk margin. Other assumptions include:

- Pr(A1) = Pr(A2) = 1/2
- Pr(B1) = 1/400
- Pr(B2) = 199/400
- Pr(B3) = 200/400
- B1 only occurs above the 1:200 balance sheet position (the SCR)
- Discounting and the risk margin are ignored
- The payment pattern is:

Year	1	2
%	0%	100%

The binary event arises from bound exposure, so a liability and matching asset of £1m must be held on the balance sheet at T0.

First consider the binary event contribution to the balance sheet position in the mean scenario. Since the amount held at T0 equals the expected result, there is no profit or loss at the end of Year 2 (ultimate) from the binary event.

Period/item	Assets @ start of period	Underwriting cashflows	Assets @ end of period	**Balance sheet contribution**
Pre-risk horizon: amounts already recognised				
Balance sheet position @ T0		**1.00**		**1.00**
Risk horizon				
Cashflows occurring				
Year 1	1.00	0.00	1.00	
Year 2	1.00	1.00	0.00	
TOTAL		1.00		1.00
Provision for future cashflows on claims incurred		0.00		0.00
Post-risk horizon				
Provision for future cashflows on unexpired risk		0.00		0.00
Change in balance sheet during risk horizon		**1.00**		**1.00**
Balance sheet position @ end of risk horizon		**0.00**		**0.00**

Next consider the contribution to the 1:200 outcome for the balance sheet position distribution (i.e. the SCR outcome). As noted, the dependency between B1 and the other risks is such that B1 occurs above the 1:200 of the balance sheet position. There is no binary event claim at the 1:200. The 1:200 outcome is therefore a profit release of £1m.

Period/item	Assets @ start of period	Underwriting cashflows	Assets @ end of period	**Balance sheet contribution**
Pre-risk horizon: amounts already recognised				
Balance sheet position @ T0		**1.00**		**1.00**
Risk horizon				
Cashflows occurring				
Year 1	1.00	0.00	1.00	
Year 2	1.00	0.00	1.00	
TOTAL		0.00		0.00
Provision for future cashflows on claims incurred		0.00		0.00
Post-risk horizon				
Provision for future cashflows on unexpired risk		0.00		0.00
Change in balance sheet during risk horizon		**0.00**		**0.00**
Balance sheet position @ end of risk horizon		**-1.00**		**-1.00**

If the above example were re-worked assuming that the binary event was the only event the syndicate was exposed to, then the technical provisions would be £1m plus the associated risk margin; the 1:200 capital requirement would be (£1m) less the risk margin and the capital stack would be nil. This makes sense, since below the 1:200 there would be no claim to be paid. If capital were set to cover all possible outcomes, then the second table would show a nil outcome, as in the mean scenario. The capital would be nil less the risk margin and the capital stack would be £1m, which would be sufficient to cover all possible outcomes on an ultimate basis.

No adjustment to the SCR is required to realise the benefit from the binary events more extreme than the 1:200; as the example shows, it is produced implicitly in the model. This is distinct from the risk margin, which is established in addition to the best estimate liabilities for the purpose of transferring the liabilities to a prospective buyer on a one year horizon.

Similarly, for reserve risk, no further adjustment is required (assuming that the binary event relates to claims provisions).

Event	Probability	Cumulative Probability	Reserve risk
B1	1/400	1.0000	399
B2	199/400	0.9975	-1
B3	200/400	0.5000	-1
Mean			0
1:200			-1

Continuing with the assumption that the binary event is the only event to which the syndicate is exposed, it can be seen that the reserve risk outcomes are consistent with the SCR. (This is to be expected since the assets are equal to the best estimate liabilities plus risk margin.) The mean reserve risk is nil and the 1:200 is (£1m). These values will equal to the SCR, after deduction of the risk margin.

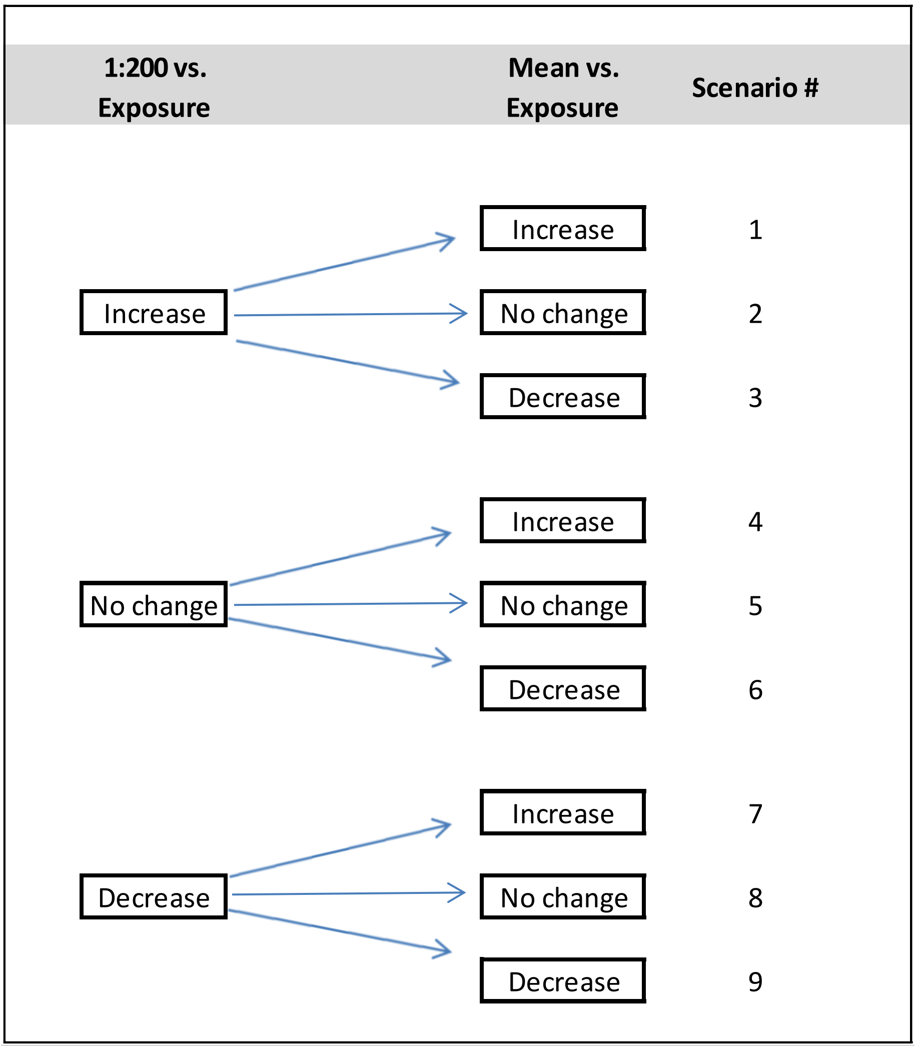
ANALYSIS OF CHANGE: HIGHLIGHTED MOVEMENTS

Background

Lloyd's reviews of the September submissions for the 2017 YOA will include an analysis of change ("AOC") against the latest submitted LCR for the 2016 YOA, plus any loadings. The AOC will be applied to the form 309 risks (excluding other credit). Significant movements in absolute (GBP) amounts or relative to exposure (premiums, reserves or a combination of them) in the mean and/or 1:200 will be highlighted for further investigation. In addition, movements in the one year vs. ultimate mean and SCR will also be flagged.

The starting point for the AOC review is that the agreed SCR for the prior YOA was based on appropriate levels for the form 309 risk types. If the risk vs. reward relationship has not changed, a movement in the 1:200 relative to exposure should be accompanied by a change in mean profit in the same direction (up or down). (The magnitude of the changes at the 1:200 and mean will not be the same in most cases.) This will normally be the outcome for most types of changes in exposure and/or the Lloyd's mandated FX rates. However the risk vs. reward relationship will be altered by changes in: (1) the shape of the underlying profit and loss distributions (before adjustments for exposure or FX); (2) the dependencies between these distributions; and (3) the allocation of profit and loss between the balance sheet and capital. Examples of both types of changes are given below for premium risk.

The possible combinations of changes in the 1:200 and mean vs. exposure are shown in the schematic below. From the standpoint of capital adequacy, scenarios 4, 7 and 8 are of the greatest interest. These involve a change in mean profit that is more favourable than the change in 1:200. Lloyd's will focus its reviews on scenarios of these types.



The AOC exhibit displays the following messages as a way of identifying potential changes in the risk vs. reward relationship. For the 2017 YOA, significant movements have been defined as +/- 5% or more vs. exposure, while “no change” has been defined as less than +/- 1% vs. exposure.

Metric	Message	Trigger	Purpose
Mean (GBP)	“The 1:200 has decreased but the mean profit has not.”	The 1:200 vs. exposure decreases by at least 5% but the mean vs. exposure changes by less than 1%	Consistency of direction of change
Mean (GBP)	“The 1:200 has not increased but the mean profit has.”	The 1:200 vs. exposure changes by less than 1% but the mean vs. exposure increases by at least 5%	Consistency of direction of change
Mean (vs. Exposure)	“The mean vs. exposure has increased/decreased by more than 5%.”	The mean vs. exposure has increased/decreased by more than 5%	Magnitude of change
1:200 (vs. Exposure)	“The 1:200 vs. exposure has increased/decreased by more than 5%.”	The 1:200 vs. exposure has increased/decreased by more than 5%	Magnitude of change

The triggers are a prompt for further investigation; exceeding them does not imply that the change cannot occur for valid reasons. Similarly, changes below the trigger may also be challenged. The triggers are intended to bring consistency and transparency to the reviews; they may however be revised by Lloyd's over time.

Premium risk AOC

There possible reasons for changes in the 1:200 risk and/or mean profit are more numerous for premium risk than other LCR risk types. The following table summarises some of the more frequently encountered examples; the list is not exhaustive.

Premium Risk: Examples of Drivers of Change		
	Expected Risk-Reward Impact (GBP)	
Change vs. Prior Year	**Mean Profit**	**1:200 Risk**
Changes in Exposure		
Increase/decrease in premium on new business (unbound) – existing classes only	Increase / decrease	Increase / decrease
Increase in premium on new business (unbound) – new classes uncorrelated with existing business	Increase	Small increase / decrease
Increase/decrease in new business (ULO) bound as at 31-Dec	No change	Increase / decrease
Increase/decrease in UPR for current YOA as at 31-Dec	No change	Increase / decrease
Increase/decrease reinsurance	Decrease / increase	Decrease / increase
Changes in FX		
Increase/decrease GBP:USD	Decrease / increase	Decrease / increase
Changes in underlying Risk vs. Reward Assumption		
Increase/Decrease in Standard Deviation or COV	No change	Increase / decrease
Increase/Decrease in inter-class dependencies	No change	Increase / decrease

An increase in planned new business for one or more existing classes, which are not considered bound at 31 December of the current year, will result in an increase in premium risk mean profit (assuming that the business is considered profitable). The increase in mean profit will be equal to the expected profit on the new business. The 1:200 should also increase, but by an amount less than the 1:200 of the additional business, due to diversification between classes. Conversely, a decrease in premium compared to the current YOA would reduce mean profit and the 1:200. In cases comparable to this scenario, Lloyd's would normally require evidence that the increase/decrease in the 1:200 is appropriate and that dependencies between classes remain sufficient; in addition, the usual requirement that the mean profit is consistent with SBF assumptions would apply.

The impact on premium risk relative to premium will vary depending on the classes for which the premium increased, and on the dependencies between classes. If there is positive dependence between classes, an increase for only one class should move the aggregate ratio closer to that of the classes for which the premium has increased. If premium has increased for a class with a lower 1:200 to premium than last year's for total premium risk, then the ratio of premium risk to premium would be expected to decrease, all else being equal. The converse would apply if premium decreased for this class. An across the board increase (decrease) for all classes should not change the mean and 1:200 vs. premium.

In the second scenario, the risk from the new class is also unbound at 31 December, but is considered uncorrelated with existing business. A simplistic example would be a syndicate writing only catastrophe business, but which plans to write a new casualty book in the prospective YOA. Suppose that the 1:200 of the catastrophe book is much larger than that of the casualty book; that the probability of making a profit on the casualty book is 75%; and that the two portfolios are considered independent. In this case it is more likely that a profit will be made on the casualty book, even at extreme percentiles on the catastrophe book; this would result in a *lower* simulated 1:200 for the portfolio as a whole. Lloyd's has however consistently adopted the position that taking on additional risk must not result in a lower capital requirement. The purpose of this position is to prevent capital drift. An AOC showing an increase in mean profit and a decrease in the 1:200 and due to lack of dependence between existing and new classes of business would be challenged by Lloyd's. One possible outcome would be for Lloyd's to require an increase in dependencies between the new and existing classes, in order to result in a level of premium risk vs. premium consistent with at least a non-nil contribution from each class. Agents would also be expected to provide support for the dependency assumptions.

In the third and fourth scenarios, the changes relate to ULOs for the prospective YOA and unexpired risk for the current YOA. These are both treated as bound business and included in the technical provisions on the starting balance sheet. The expected profit for this business would be booked on the opening Solvency II balance sheet; it would not contribute to premium risk mean profit. The risk on this business would however contribute to the 1:200. If for example the planned premium from ULO business were to be reduced compared to the previous year, the premum risk mean profit would remain unchanged (all else being equal), but the 1:200 would decrease. Similarly, if ULOs were to increase, the mean would remain unchanged and the 1:200 should increase.

A different scenario involves the reclassification of ULO business as non-ULO (i.e. unbound) as at 31 December or T0. The profit on the re-classified business would move from the balance sheet to capital (premium risk); however the 1:200 would decrease, since additional capital would no longer be required to cover the credit taken on the balance sheet. The stress (1:200 less mean) would remain unchanged; so would the total resource requirement or capital stack.

A reduction in 1:200 without a corresponding decrease in mean profit could also occur with a change in UPR. In a steady state scenario, if less premium is written in the current YOA than had been planned for, the UPR included in the SCR will be reduced relative to the previous year's. The reduction in UPR would not reduce premium risk mean profit; it would however reduce the 1:200. A similar result could occur due to an acceleration of the earning pattern; in this case an increase in reserve risk would also be expected, assuming no change in total premium.

In cases such as these relating to changes in bound business, agents should be prepared to provide evidence that the changes in risk vs. reward are due to movements in exposure and/or classification, not the underlying profit and loss distribution for classes of business. For a more detailed discussion on the calculation of premium risk on bound vs. unbound business, please see the SCR guidance numerical examples on lloyds.com.

The final example of changes in exposure relates to reinsurance (a change to net exposure). Additional reinsurance should reduce the 1:200 but also reduce the mean profit. The impact will vary depending on the type of cover and market conditions. RI credit risk should increase in GBP; significant changes relative to expected recoveries will be challenged.

Movements due to changes in FX rates should normally be small relative to exposure, since the FX rates will impact both claims and premiums/reserves. Claims however may be paid over a much longer time period, allowing for more variation in FX.

Finally, the two examples of changes in risk vs. reward will not impact the mean but will change the 1:200. These changes are based on an altered view of risk. As noted above, Lloyd’s will challenge the basis for such changes.

Other risk types

The same principle of risk vs. reward will be used in evaluating the AOC for the other LCR risk types. The AOC for these risks will normally be easier to evaluate, since issues relating to the classification of business as bound vs. unbound will not apply.

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